SEMESTER-III

COURSE 8: FUNCTIONS OF A COMPLEX VARIABLE

Theory Credits: 4 5 hrs/week

Course Outcomes

After successful completion of this course, the student will be able to

1. determine a Bilinear transformation under given condition
2. know about continuity, compactness and connectedness of sets in complex plane
3. know the necessary condition and sufficient condition for f(z)to be analytic
4. know about the inverse of an analytic function
5. know about the convergence of sequences and the necessary & sufficient condition for a sequence to be convergent
6. know the power series expansion of elementary functions

Course Content

Unit – 1
Bilinear Transformations

Extended Complex Plane – Resultant and Inverse of a bilinear transformation – The linear group – Geometrical significance of the transformation. Angle preserving property of Bilinear Transformation– Determination of Bilinear transformations under given condition, some special bilinear transformations.

Unit – 2
Topological Considerations

Neighbourhood of a point – Interior, exterior and frontier points of a set, open and closed sets. Connected sets, Domains and continua - a theorem on Nests of closed Rectangular domains- Bolzano Weierstrass theorem- Hein-Borel theorem. Limits - algebraic operations with limits – continuity and uniform continuity – compactness – connectedness - Jordan curve theorem - connectedness of line segments and polygonal lines. Branch line and Branch point - Characterisation of open connected sets by polygonal lines.

Unit – 3
Analytic functions

Differentiable functions of a complex variable - Geometrical representation of a variable - Analytic function- Elementary rules and chain rule - Derivatives of polynomials and rational functions - The necessary condition and sufficient condition for $f(z)$to be analytic - Analytic functions in a Domain – Derivative of w in polar form - Construction of $f(z)$.

Unit – 4
Inverse of an analytic function and infinite series

The inverse of an analytic function – neighbourhood preserving mappings - Domain preserving and angle preserving property of analytic mappings.

Convergent sequences, necessary and sufficient condition for a sequence to be convergent, Cauchy sequence, Convergence of infinite series. Cauchy general principle of convergence for a series. Absolute convergence of a series. Abel's and Dirichilet's tests. Rearrangement of series, product of series.

Unit – 5
Power Series

Power series - exponential, trigonometric and hyperbolic functions - zeros of $sin\ z, cos\ z$ - periods of $sin\ z, cos\ z, E(z)$ - A law of logarithms - Analytic character of $log\ z$ - generalized a^b - Analytic character of z^n - $\mathrm{Cos}^{-1}\ z$, $\mathrm{Sin}^{-1}\ z$ and derivatives of $\mathrm{Cos}^{-1}\ z$, $\mathrm{Sin}^{-1}\ z$.

Activities

Seminar/ Quiz/ Assignments/ Applications of Functions of complex variables to Real life Problem /Problem Solving Sessions.

Text Book

Theory of Functions of a Complex variable by Shanti Narayan &Dr. P. K. Mittal, S. Chand &Company Ltd.

Reference Books

1. Theory of Functions of a Complex Variable by A. I. Markushevich, Second Edition, AMS Chelsea Publishing
2. Theory And Applications by M. S. Kasara, Complex Variables, 2nd Edition, Prentice Hall India Learning Private Limited

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